Discussion of “Assessing Stage-Discharge Relationships for Circular Overflow Structure” by M. Bijankhan and V. Ferro

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Introduction
The discusser would like to thank the authors for using power-law functions for calculating the flow discharge through vertical circular weirs. The discusser, however, would like to draw attention to two regression coefficients that should be determined by experimental data.

Incomplete Self-Similarity Condition
Since head ratio, \( h/D \), is always less than unity (Fig. 1) and cannot approach infinity from the hydraulic viewpoint, the proposed stage-discharge relationship by the authors [Eq. (14) of the original paper] cannot be deduced by self-similarity theory, and thus this relation has no theoretical base. The proposed relationship is a regression-based one, which is different from the theoretical-based weir stage-discharge equation, as will be shown.

Theoretical Considerations
Fig. 1 shows a circular weir. The head-discharge relationship of a sharp-crested circular weir can be derived using the energy equation for a horizontal elemental strip of the flow as follows (Vatankhah 2010, 2016):

\[
Q = 2C_d \sqrt{2gD^3} \left( \frac{h}{D} \right)^2 \int_0^1 \sqrt{x(1-x)} \left( 1 - x \frac{h}{D} \right) dx
\]  
(1)

where \( g \) = gravitational acceleration; \( h \) = upstream water depth relative to the weir crest; \( Q \) = weir discharge; and \( D \) = weir diameter.

Eq. (1) is valid over the practical range \( 0 < h/D < 1 \). The flow discharge coefficient, \( C_d \), is introduced to correct the nonrealistic assumptions such as effects of approach velocity, viscous effects, and streamline curvature due to the weir contraction (Vatankhah and Khalili 2017). Due to the fluid resistance, the actual discharge is expected to be less than the theoretical discharge. Thus, the discharge coefficient, \( C_d \), should be less than unity for the standard weir discharge equations in the form of \( Q = C_d Q_t = C_d \int V dA \), in which \( Q_t \) is the theoretical discharge, \( V \) is the flow velocity of the elemental strip, and \( dA \) is the area of the elemental strip.

The integral term in Eq. (1) can be approximated using the method of undetermined coefficients (Vatankhah 2011). The final result can be expressed as (Vatankhah 2016)

\[
Q = 0.79C_d \sqrt{2gD^3} \left( \frac{h}{D} \right)^2 \sqrt{1 - 0.54 \frac{h}{D}}
\]  
(2)

The stage-discharge Eq. (2) is theoretically deduced irrespective of \( C_d \) variations.

A simple power-law regression function can be used for the discharge coefficient as follows:

\[
C_d = a \left( \frac{h}{D} \right)^b
\]  
(3)

where \( a \) and \( b = \) regression coefficients that should be determined by experimental data.

Substituting Eq. (3) into Eq. (2) yields the following simple equation for actual discharge, \( Q \), of the sharp-crested circular weir:

\[
Q = k \sqrt{gD^3} \left( \frac{h}{D} \right)^m \sqrt{1 - 0.54 \frac{h}{D}}
\]  
(4)

where \( k \) and \( m \) = two regression coefficients that should be determined by experimental data. As noted from Eq. (4), the stage-discharge equation of the circular weirs cannot be solely expressed using a power-law regression equation and a simple radical term is also required.

In Eq. (4), the upstream head is considered as an independent variable. To find the upstream head for a given design discharge, Eq. (4) can be solved using the fixed-point method as

\[
\frac{h}{D} = \left( k \sqrt{1 - 0.54 \frac{h}{D}} \right)^{-1/m} \left( \frac{Q}{\sqrt{gD^3}} \right)^{1/m}
\]  
(5)

Expanding radical term of Eq. (5) and using two first terms yields

\[
\frac{h}{D} = k^{-1/m} \left( 1 + 0.27 \frac{h}{m} \right) \left( \frac{Q}{\sqrt{gD^3}} \right)^{1/m}
\]  
(6)

By applying \( h/D = a < 1 \) in the right-hand side of Eq. (6), an initial guess can be obtained as

![Fig. 1. Cross section of a fully contracted sharp-crested circular weir (0 < h/D < 1).](https://example.com/image.png)
Substituting Eq. (7) into Eq. (6) yields the following simple equation for the head ratio:

\[
\frac{h}{D} = k^{-1/m} \left( 1 + \frac{0.27m}{Q\sqrt{gD^5}} \right)^{1/m}
\]

Eq. (8) can be expressed as

\[
\frac{h}{D} = a \left( \frac{Q}{\sqrt{gD^5}} \right)^n + b \left( \frac{Q}{\sqrt{gD^5}} \right)^{2n}
\]

where \(a\), \(b\), and \(n\) are three regression coefficients that should be determined by experimental data.

Using all data sets reported in the original paper (333 runs; the flow head data for \(D = 0.6858\) m were incorrectly reported in the original paper and should be corrected), the proposed Eq. (4) and its inverse [Eq. (9)] are calibrated. The final results are as follows:

\[
Q = 0.65 \sqrt{gD^5} \left( \frac{h}{D} \right)^{1.985} \sqrt{1 - 0.54 \frac{h}{D}} \tag{10}
\]

\[
\frac{h}{D} = 1.2 \left( \frac{Q}{\sqrt{gD^5}} \right)^{0.5} + 0.376 \left( \frac{Q}{\sqrt{gD^5}} \right) \tag{11}
\]

Eq. (10) has an average error of 3.5%, and 81% of the relative errors are within ±5% (93% of the relative errors are within ±10%). Similarly, Eq. (11) has an average error of 2%, and 92% of the relative errors are within ±5% (97% of the relative errors are within ±10%). By excluding Cone’s data set for \(D = 0.914\) m (with high errors and different unknown behavior), Eq. (10) will have an average error of 2.9%, while Eq. (11) will have an average error of 1.75%.

**Conclusions**

The theoretical stage-discharge equation obtained by integration of the velocity components over the circular weir section was simplified, then a power-law function was used for the discharge coefficient to get a simple stage-discharge relationship with only two calibration coefficients [Eq. (4)]. To find the upstream head for a given design discharge, Eq. (4) was inverted as Eq. (9) with three calibration coefficients. Both proposed equations [Eqs. (4) and (9)] are simple and can be calibrated using any data sets. The simple proposed equations were calibrated using all data sets reported in the original paper (333 runs) as Eqs. (10) and (11), which are very useful for practical purposes.

**References**


